

Workshop for Economic Modeling in Julia

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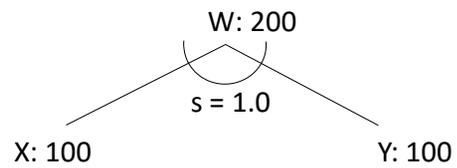
Intermediate input and Nesting

- M22.gms (Markusen, 2002)

Production Sectors			Consumers	
Markets	X	Y	W	CONS
PX	120	-20	-100	
PY	-20	120	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

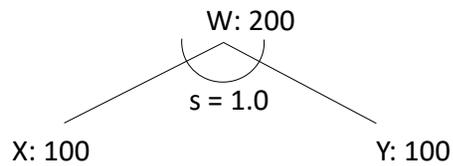
Intermediate input and Nesting

- Production structure of W



Intermediate input and Nesting

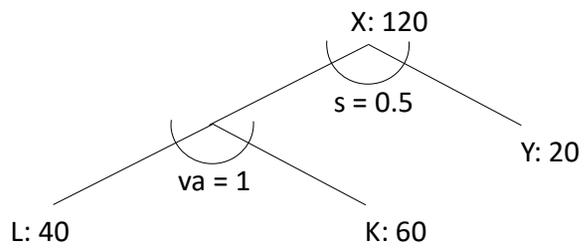
- Production structure of W



```
@production(M22, W, [s = 1,t = 0], begin
  @output(PW, 200, t)
  @input(PX, 100, s)
  @input(PY, 100, s)
end)
```

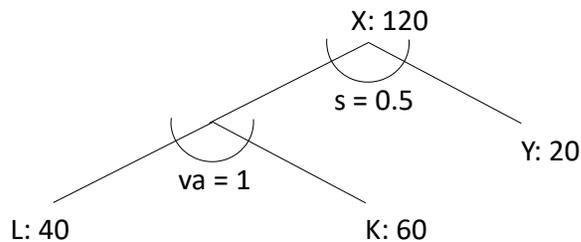
Intermediate input and Nesting

- Production structure of X



Intermediate input and Nesting

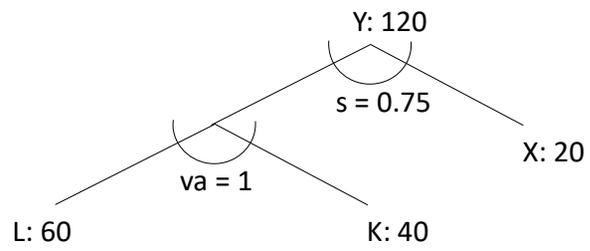
- Production structure of X



```
@production(M22, X, [s = 0.5, t = 0, va => s = 1], begin
  @output(PX, 120, t)
  @input(PY, 20, s)
  @input(PL, 40, va, taxes = [Tax(CONS, TX)])
  @input(PK, 60, va, taxes = [Tax(CONS, TX)])
end)
```

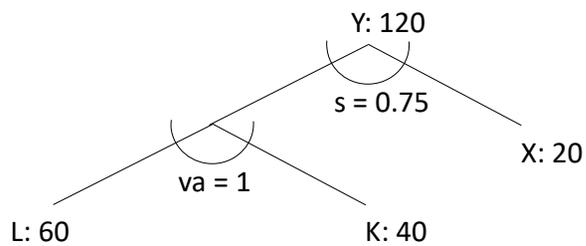
Intermediate input and Nesting

- Production structure of Y



Intermediate input and Nesting

- Production structure of Y



```
@production(M22, Y, [s = 0.75, t = 0, va => s = 1], begin
  @output(PY, 120, t)
  @input(PX, 20, s)
  @input(PL, 60, va)
  @input(PK, 40, va)
end)
```

Auxiliary variable

- M33.gms (Markusen, 2002)
 - Positive labor supply curve with work vs. leisure choice
 - 20% benchmark tax rates on labor and capital inputs
 - Change labor tax rate and find the capital tax rate that keeps the total tax revenue “constant”
 - Constant tax revenue means: after the change, the consumer can still afford the benchmark consumption bundle

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors			TL	TK	Consumers CONS
	A	B	W			
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors			Consumers		
	A	B	W	TL	TK	CONS
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors			Consumers		
	A	B	W	TL	TK	CONS
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors					Consumers
	A	B	W	TL	TK	CONS
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors					Consumers
	A	B	W	TL	TK	CONS
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40



PLS = 1.2



PKS = 1.2



PL = 1.0



PK = 1.0

Auxiliary variable

- M33.gms (Markusen, 2002)

Markets	Production Sectors					Consumers
	A	B	W	TL	TK	CONS
PX	120		-120			
PY		120	-120			
PW			340			-340
PLS	-48	-72		120		
PKS	-72	-48			120	
PL			-100	-100		200
PK					-100	100
TAX				-20	-20	40



PLS = 1.2



PKS = 1.2



PL = 1.0



PK = 1.0

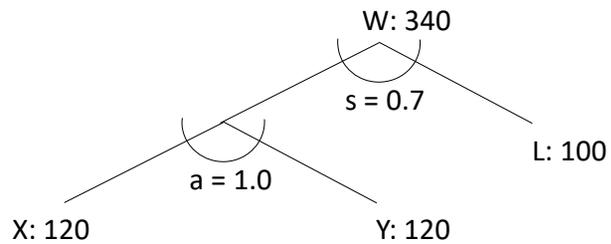
Auxiliary variable

- Declaration for the auxiliary variable

```
@auxiliaries(M33, begin  
|   TXK  
end) Endogenous
```

Auxiliary variable

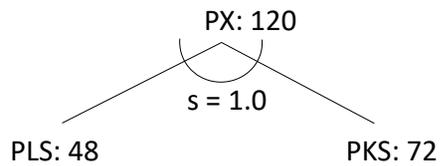
- Production structure of W



```
@production(M33, W, [t = 0, s = 0.7, a => s = 1], begin
  @output(PW, 340, t)
  @input(PX, 120, a)
  @input(PY, 120, a)
  @input(PL, 100, s)
end)
```

Auxiliary variable

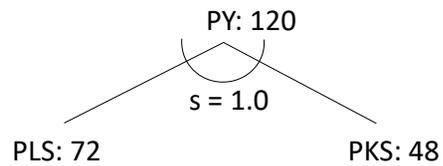
- Production of X



```
@production(M33, X, [t = 0, s = 1], begin
  @output(PX, 120, t)
  @input(PLS, 40, s, reference_price = 1.2)
  @input(PKS, 60, s, reference_price = 1.2)
end)
```

Auxiliary variable

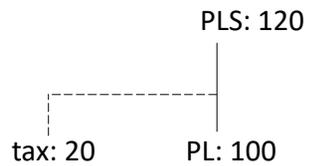
- Production of Y



```
@production(M33, Y, [t = 0, s = 1], begin
  @output(PY, 120, t)
  @input(PLS, 60, s, reference_price = 1.2)
  @input(PKS, 40, s, reference_price = 1.2)
end)
```

Auxiliary variable

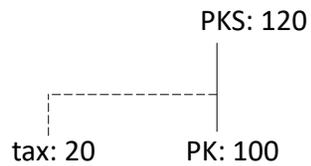
- Production of tax-included L



```
@production(M33, TL, [t = 0, s = 0], begin
|   @output(PLS, 100, t, reference_price = 1.2)
|   @input(PL, 100, s, taxes = [Tax(CONS, TXL)])
end)
```

Auxiliary variable

- Production of tax-included K



```
@production(M33, TK, [t = 0, s = 0], begin
  @output(PKS, 100, t, reference_price = 1.2)
  @input(PK, 100, s, taxes = [Tax(CONS, TXK)])
end)
```

Auxiliary variable

- Constraint equation for “equal-yield tax revenue”

```
@aux_constraint(M33, TXK, begin  
|   TXL*PL*TL*100 + TXK*PK*TK*100 - 40*(PX + PY)/2  
end)
```

Auxiliary variable

- Constraint equation for “equal-yield tax revenue”

```
@aux_constraint(M33, TXK, begin
| TXL*PL*TL*100 + TXK*PK*TK*100 - 40*(PX + PY)/2
end)
```

Endogenous

Exogenous

Tax revenue under the benchmark consumption bundle but with variable prices

Project

- Turn the following models in Markusen's tutorial to Julia versions
 - M24 Introduces the use of specific factor
 - M29 Introduces Stone-Geary preferences
 - M31 Closed economy 2x2 with taxes in the benchmark
 - M36 Closed economy 2x2, taxes and classical unemployment

Bibliography

- Markusen, J. (2002). General-Equilibrium Modeling using GAMS and MPS/GE: Some Basics. University of Colorado, Boulder. <https://www.mpsge.org/tutorial.pdf>
- Rutherford, T. (2002). Lecture Notes on Constant Elasticity Functions. University of Colorado, Boulder. https://downloads.regulations.gov/EPA-HQ-OAR-2022-0730-0088/attachment_58.pdf